EE 232: Lightwave Devices
Lecture #13 – Absorption and gain in quantum dots

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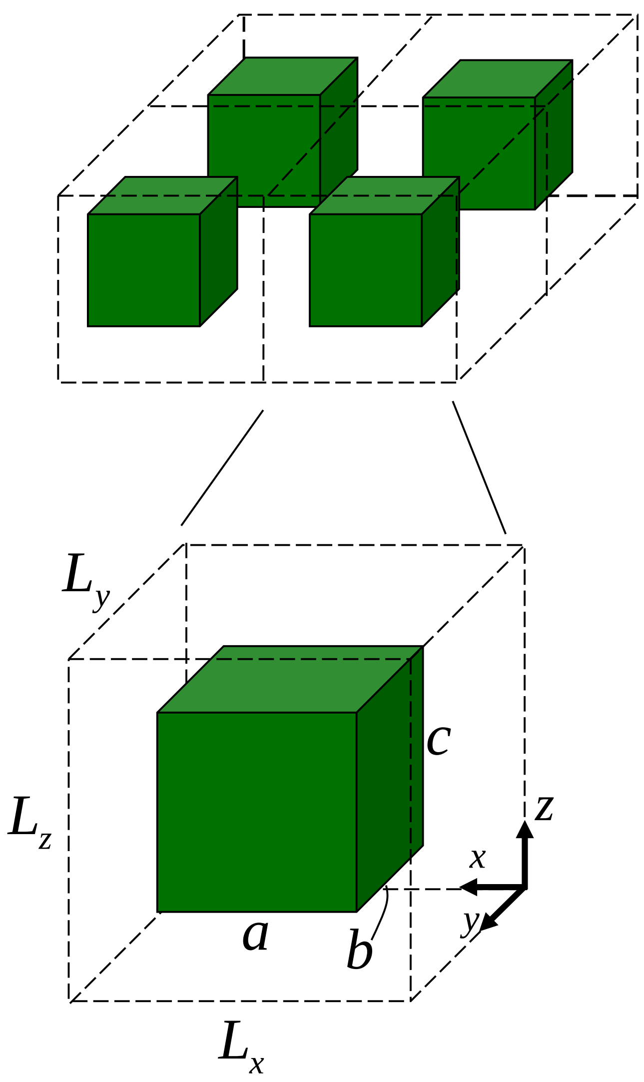
Quantum dot - simple model



Geometry

Assume infinite plane of box-shaped semiconductor quantum dots embedded in a matrix of another semiconductor.

Quantum dot volume: $V_{QD} = abc$

Wavefunctions and energies (infinite barrier model)

$$\psi_c = \frac{\sqrt{8}}{\sqrt{abc}} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{l\pi z}{c}\right) u_c(x,y,z)$$

$$\psi_v = \frac{\sqrt{8}}{\sqrt{abc}} \sin\left(\frac{m'\pi x}{a}\right) \sin\left(\frac{n'\pi y}{b}\right) \sin\left(\frac{l'\pi z}{c}\right) u_v(x,y,z)$$

$$E_e^{mnl} = E_g + \frac{\hbar^2}{2m_e^*}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2\right]$$

n, m, l positive integer

$$E_h^{m'n'l'} = \frac{-\hbar^2}{2m_h^*}\left[\left(\frac{m'\pi}{a}\right)^2 + \left(\frac{n'\pi}{b}\right)^2 + \left(\frac{l'\pi}{c}\right)^2\right]$$

n', m', l' positive integer

Absorption coefficient

$$\alpha(\hbar\omega) = C_0 \frac{2}{V} \sum_{k_v} \sum_{k_c} \left| \langle \psi_c | \hat{e}\cdot\mathbf{p} | \psi_v \rangle \right|^2 \delta(E_e - E_h - \hbar\omega)(f_v - f_c)$$

$$= C_0 \frac{2N_{QD}}{V} \sum_{mnl} \left| \hat{e}\cdot\mathbf{p}_{cv} \right|^2 \delta(E_e^{mnl} - E_h^{mnl} - \hbar\omega)(f_v - f_c)$$

$$= C_0 \frac{2N_{2D}}{L_z} \sum_{mnl} \left| \hat{e}\cdot\mathbf{p}_{cv} \right|^2 \delta(E_e^{mnl} - E_h^{mnl} - \hbar\omega)(f_v - f_c)$$

$$N_{2D} = \frac{1}{L_x L_y}$$ (areal dot density)

$$\left| \hat{e}\cdot\mathbf{p}_{cv} \right|^2 = \left| \langle u_c | \hat{e}\cdot\mathbf{p} | u_v \rangle \right|^2 \delta_{mm'}\delta_{nn'}\delta_{ll'}$$

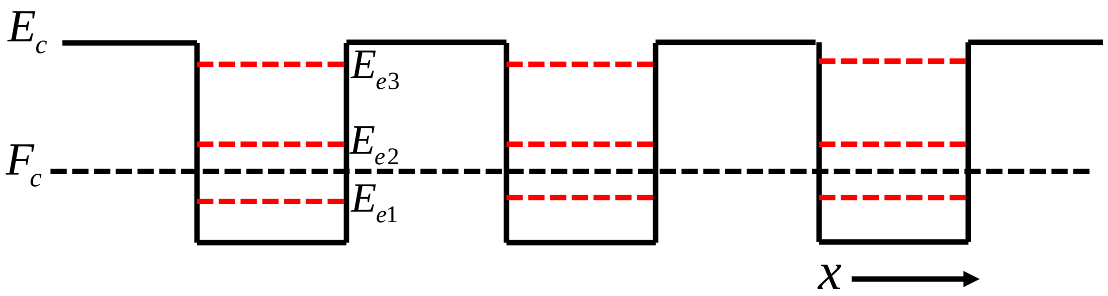
Including intraband scattering processes

$$\alpha(\hbar\omega) = C_0 \frac{2N_{2D}}{L_z} \sum_{mnl} \left| \hat{e}\cdot\mathbf{p}_{cv} \right|^2 L(E_{eh}^{mnl} - \hbar\omega)(f_v - f_c)$$

$$L(E - \hbar\omega) = \frac{\gamma/\pi}{(E - \hbar\omega)^2 + \gamma^2}$$

Carrier density

$$n = \text{Carrier density} = \frac{\text{number of states per quantum dot} \times \text{probability the states are filled}}{\text{volume of quantum dot layer}}$$



Quasi-Fermi energy is assumed to be constant across the quantum dot layer

$$n = N_{QD} \frac{2}{V} \sum_{mnl} f_c(E)$$

$$= 2 \frac{N_{2D}}{L_z} \sum_{mnl} f_c(E)$$

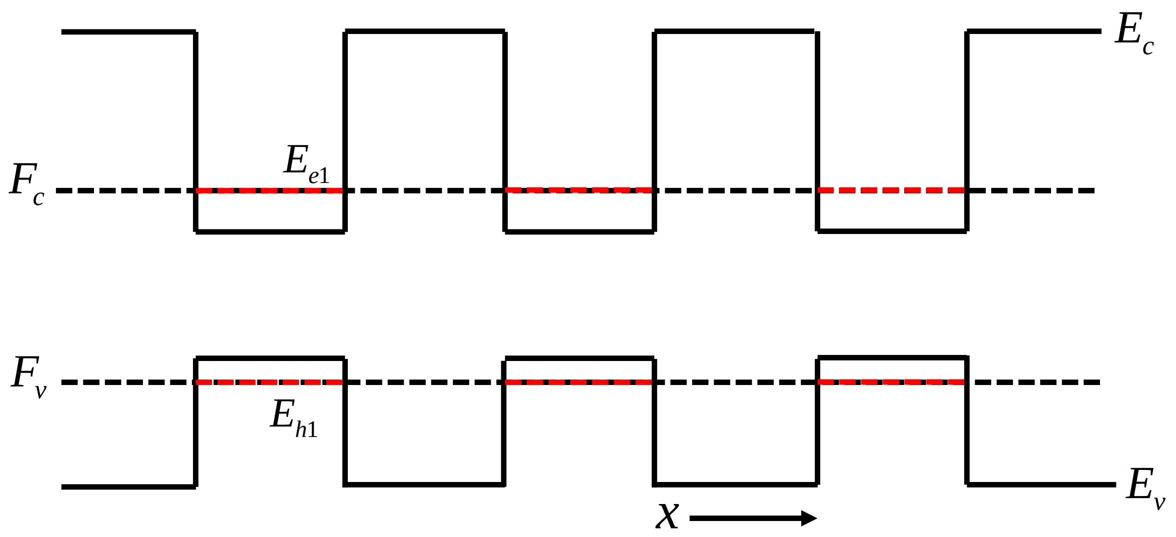
$$n = 2 \frac{N_{2D}}{L_z} \sum_{mnl} \frac{1}{1 + e^{(E_g + E_e^{mnl} - F_c)/kT}}$$

Similarly,

$$p = 2 \frac{N_{2D}}{L_z} \sum_{mnl} \frac{1}{1 + e^{(F_v - E_h^{mnl})/kT}}$$

Transparency carrier density

Assuming only the ground state is filled

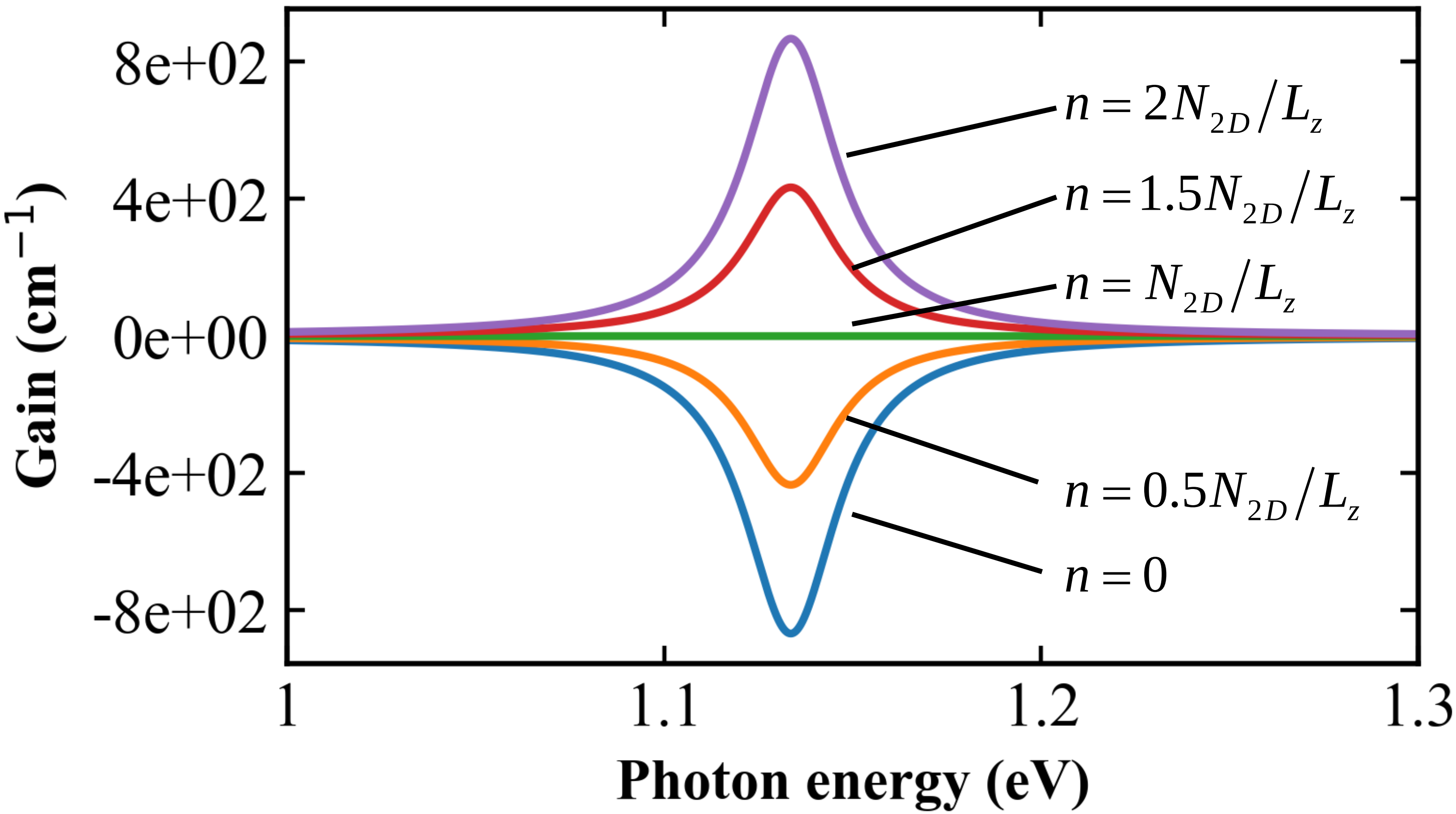


$$F_c - F_v = E_g + E_{e1} - E_{h1}$$ (transparency condition)

Quasi Fermi-Level positions at the transparency condition will be at the eigenergies of the ground state. This implies that half of those eigenstates are filled.
For each state, there are two possible electrons that can fill that state (factor two from spin)
Then, trivially,

$$n_{th} = \frac{N_{2D}}{L_z}$$

Absorption / Gain spectrum



InAs quantum dot $(N_{2D} = 5\times10^{10}\,\mathrm{cm}^{-2})$

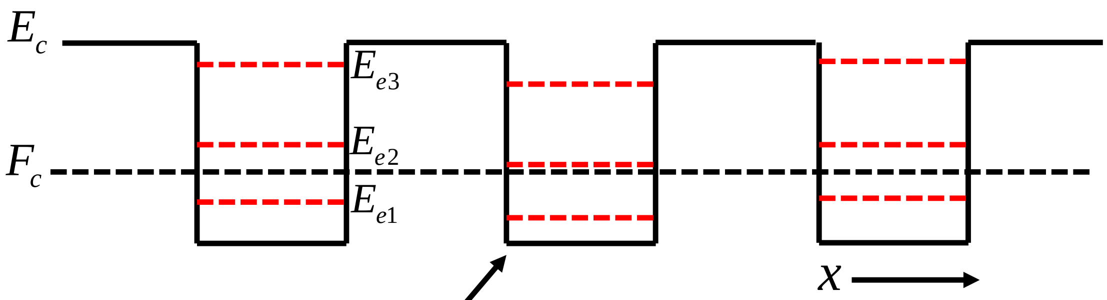
Consider only the ground state populated

Use bulk matrix element

Include only homogenous broadening $(\gamma = 15\ \mathrm{meV})$

Inhomogeneous broadening

Quantum dots are not uniformly sized therefore the eigenenergies of each quantum dot state will vary about some overall average. The probability that a state will be filled will vary from dot-to-dot. This is taken into account by weighting the Fermi function with a Gaussian function.



Quasi-Fermi energy is assumed to be constant across the quantum dot layer

More likely that state E_{e1} is filled in this quantum dot

$$n = 2\frac{N_{2D}}{L_z}\sum_{mnl} f_c(E) \rightarrow \boxed{n = 2\frac{N_{2D}}{L_z}\sum_{mnl}\int_0^{\infty} G(E) f_c(E) dE}$$

$$G(E) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-(E-E_e^{mnl})^2/2\sigma_c^2}$$

Inhomogeneous broadening

Similarly, we must broaden the Fermi functions in the absorption expression.

$$\alpha(\hbar\omega) = C_0 \frac{2N_{2D}}{L_z} \sum_{mnl} \int_0^\infty dE \left|\hat{e}\cdot\mathbf{p}_{cv}\right|^2 D(E) L(E-\hbar\omega)(f_v - f_c)$$

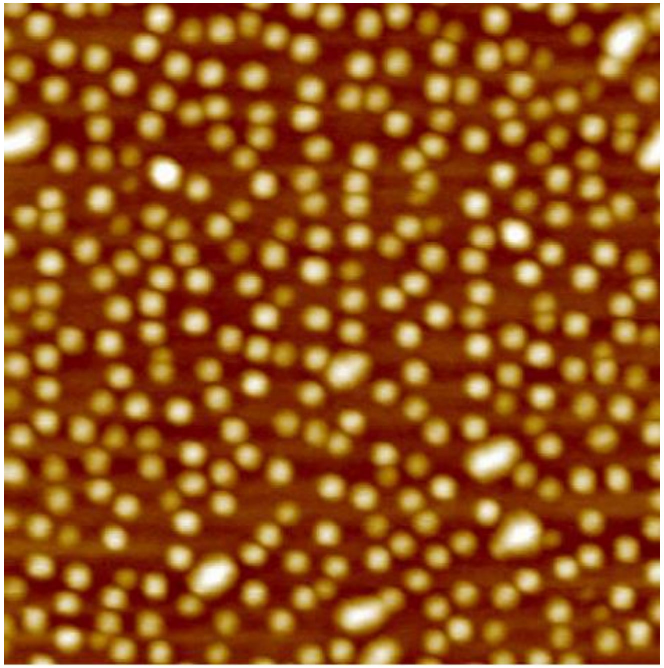
$$D(E) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(E-E_{eh}^{mnl})^2/2\sigma^2}$$

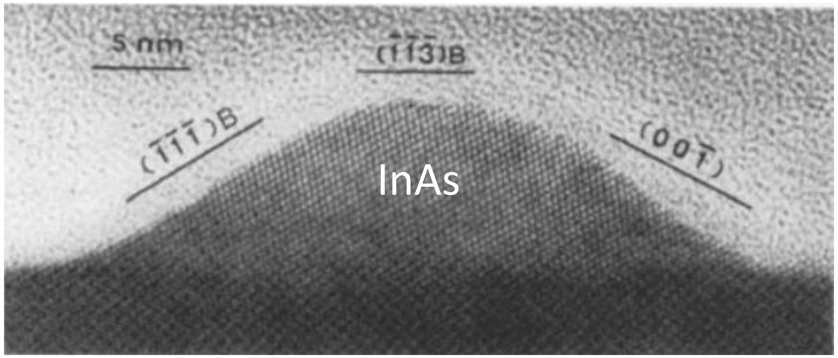
$$\sigma^2 = \sigma_e^2 + \sigma_h^2$$

electron broadening (σ_e^2) hole broadening (σ_h^2)

Epitaxially grown quantum dots

Atomic force microscope (AFM) image



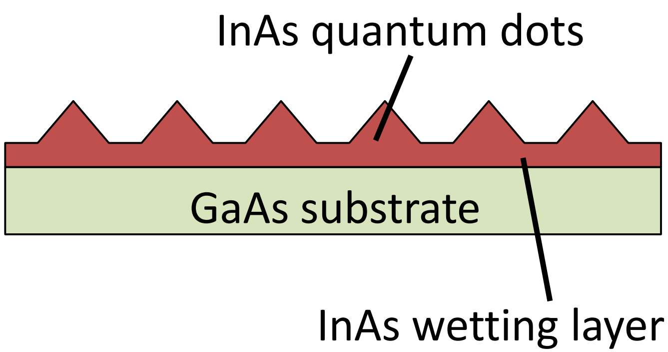


Transmission electron microscope (TEM) image

Realistic quantum dots are not box-shaped.

Shape depends upon growth conditions, materials, strain, etc.

e.g. InAs quantum dots grown on GaAs are often pyramidal in shape.



TEM image from: Y. Masumoto et al., Semiconductor Quantum Dots. Book. 2002.

AFM image from: Franchi et al. Progress in Crystal Growth and Characterization of Materials. 47 (2003) 166-195.